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A MODEL FOR FAULT PLANE WITH MANY ASPERITIES

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The breaking of a single asperity is treated as a typical subsource generating an elementary short-period radiation pulse at the source of a large earthquake. The model offered in 13 and 14 is used to derive formulas which describe short-period radiation parameters. The stress drop at an asperity was determined in several ways; all estimates were consistent and averaged a few hundred bars. The asperity size was about 1 km. A relationship was derived between this model and the source dynamics using the concept of barriers as chains of rupture-resistant asperities. The probability distribution of stress drop over an asperity was examined. The accelerogram is suggested to be considered as a sum of slightly overlapping pulses due to individual asperities.

INTRODUCTION

The study of short-period wave generation at the earthquake source is essential for a better understanding of earthquake focal mechanisms and for developing a theory for ground motion prediction. Short-period radiation (SP radiation) is not yet understood, even though it has been studied for many years [2, 5, 23]. The barrier model [12], its "specific" version [31, 32], the dynamic model [9], and other developments — all assume SP radiation to be excited by numerous small cracks or subsources separated by regions of no rupture (indestructible barriers), rather than by a single motion on the fault. This assumption is not quite satisfactory from the standpoint of tectonophysics, because in terms of geologic time movement on a fault is a monotonic and unidirectional process, so it is not clear exactly when the above barriers experience failure. As mentioned in [2], this model is also not quite adequate in terms of seismic radiation properties. For instance, the idealized spectral shape [32] is not quite consistent with the observed spectra, and the $f_2(f_{max})$ parameter [2, 20] which has to be introduced is not intrinsic to the model. For this reason the model of SP radiation as a set of numerous subsources [2] was descriptive in character and the mechanical nature of a subsource was not considered. However, the spectral shape assumed in the model for a subsource was consistent with the model of a crack. An alternative to the "specific" barrier model was the description of a stochastic source in terms of a stochastic strength field and/or a stochastic stress field [7, 8, 28, 30]*.

This paper examines the corollaries of the assumption made in [3] that the typical subsource is a rupturing asperity. The rupturing of a single asperity was discussed in [13] and extended in [14] to the case of an asperity at the center of a circular fault. Note that the model suggested here has no direct relevance to the asperity model offered in [26] where by asperity is meant a much larger body, not related to SP radiation. For clarity we will term it the model of a fault plane with many asperities and abbreviate it as the MA model.

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*No comparison is offered here between theoretical models and the fault systems observed in the meizoseismal areas of great earthquakes such as the Gobi-Altai earthquake. — Ed.

The development presented here largely follows the lines of [21] and [27] where there was a clear recognition of a direct relation between individual asperities (or stress concentrations) and the parameters of SP radiation. Similar ideas on the source structure have been recently put forward in [10].

In this paper we will use the MA model to determine source dimensions and stress drop and examine the distribution of asperities by breaking strength. The scaling law of far-field spectra and a near-source theory for the MA model will be discussed in later publications.

A SINGLE RUPTURING ASPERITY AND ITS RADIATION PARAMETERS

In this section we will discuss the concept of an asperity and summarize basic formulas for the radiation excited during its rupture, following to a considerable extent Das and Kostrov [13, 14]. But let us first touch upon the problem of why asperities need be considered in a seismic source theory.

The elastic linear compressive strain due to hydrostatic pressure ("compression") does not exceed 1 percent at the depths of shallow-focus earthquakes. We know that a flattened spheroidal cavity existing in an elastic medium under no stress and having the semiaxes ratio $\alpha \ll 1$ will close under hydrostatic pressure when the compressive strain attains a value of the order of α. A geologic fault can be assumed to be a contact of two rough planes at some typical angle of discordance β. Extrapolating the above statement for a spheroidal cavity, one can safely assume that the walls of a fault with $\beta > 0.01$ will not close at 50-100 km depths. The value $\beta = 0.01$ gives an angle of 0.6°, whereas the typical angle between the actual fault plane and the average plane (i.e., the typical plane discrepancy) is 1-3° and more. This means that the walls will be in direct contact over a small fraction of a fault plane not only at depths of a few kilometers [13], but in all shallow-focus earthquakes. It can be hypothesized that the contact will be good enough in more or less isometric patches (asperities) outside of which the contact is zero (when filled with a fluid) or low-strength (with a plastic filling).

As has been mentioned in [13], breaking of asperities by shear must necessarily take place as the sides of a fault move relative to one another and may be directly related to the earthquake source process. Where asperities are spaced far enough from one another, any of them can break nearly independently of the others. For this reason we will first, following [13], discuss some properties of the breaking of a single asperity which are important in terms of elastic wave radiation.

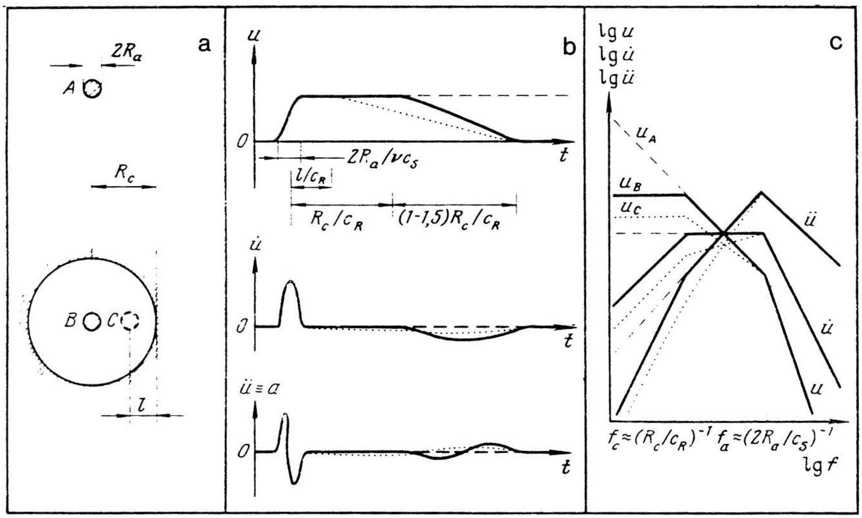

Fig. 1. Diagrammatic representation of (a) an asperity, (b) waves radiated by it, and (c) amplitude spectra of these waves for: A — an asperity consisting of two welded half-spaces, B — an asperity at the center of a circular fault (main case), and C — an asperity near the edge of a circular fault. Time functions and spectra are shown by dashes (A), solid line (B), and dotted line (C).

TABLE I

Asperity Parameter Values from Accelerograms Presented in Figure 2*

Earthquake	Station, component	r, km	u, cm	T_a, s	log F_0, dynes	R_a, km	$\Delta\tau$, bars
San-Fernando, 1971	Pacoima, top	14	35**	1.5	19.93	2.6	400
San-Fernando, 1971	Lake Hewes 4, top	25	2	1.0	18.94	1.7	90
Tokachi, 1968	Muroran, SH	160	9	0.9	20.41	2.1	1800***

Note. *Calculated from (2) and (3) taking into account twofold free surface magnification and putting c_S = 3.5 km/s, ρ = 2.7 g/cm^3, D = 1.16, and ν_a = 1.0 for San-Fernando and ν_a = 1.35 for Tokachi; ** the value of u and hence those of F_0 and $\Delta\tau$ seem to be overestimated on account of the topography; both San-Fernando estimates are for the same asperity; *** — an abnormally large pulse.

P and SH waves radiated by a single asperity (Figure 1) behave in the far-filed zones as a unidirectional velocity pulse (displacement step) whose duration (rise time of the step) T_a is approximately equal to breaking time [13]. The area of the velocity pulse is proportional to the total "seismic force of an asperity"

$$F_0 = \int_{\Sigma} \Delta\sigma(x, y)\, dS = \Delta\tau S_a, \tag{1}$$

where $\Delta\sigma = \sigma_{coh} - \sigma_{fr}$ is stress drop (strength σ_{coh} less the residual friction σ_{fr}), Σ is the asperity region, S_a its area, and $\Delta\tau$ is the mean stress drop on the asperity. The important fact is that F_0 is independent of final slip B in the place of the asperity. Disregarding rupture details and the Doppler effect, one can define the variable seismic force of a point asperity $F_0(t)$ in such a way as to have $F_0(0) = 0$, $F_0(t_a) = F_0(\infty) = F_0$. The displacement u (velocity $\dot{u}$, acceleration $\ddot{u}$) in P and SH waves is given by (for $\dot{u}$ and $\ddot{u}$ $F_0(\cdot)$ need be replaced by $\dot{F}_0(\cdot)$ or $\ddot{F}(\cdot)$)

$$u^{P,SH}(\mathbf{r}, t) = \frac{D^{P,SH} F_0(t - r/c_{P,S})}{4\pi\rho c_{P,S}^2 r}, \tag{2}$$

where $\mathbf{r}$ is the vector from source to observation site, $r = |\mathbf{r}|$, ρ is the density, $c_{P,S}$ is P,S wave velocity, $D^{P,SH}$ is the radiation pattern. We have $D^{P,SH} = D^{P,SH} R^{P,SH}$, where $R^{P,SH}$ is the standard radiation pattern for a dislocation source for the same area Σ with the normal $\mathbf{n}$ and slip direction $\mathbf{b}$ "along" $\Delta\tau$ (i.e. $b_i \sim \tau_{ij} n_j$); $D^{P,SH}$ is a "correction factor" whose value is around two. For SV waves, $\dot{u}(\mathbf{r}, t)$ will for some angles be a linear combination of $\dot{F}_0(t)$ and the Hilbert transform of $\dot{F}_0(t)$; this will not however affect the amplitude spectral shape which will in all cases be identical for SV and SH waves.

The breaking time T_a was found in [13] by a numerical experiment using a discrete model for a circular asperity of diameter $2R_a$ taken to be equal to ten grid spacings. The result was

$$T_a = 2R_a/\nu_a c_S, \tag{3}$$

with ν_a = 0.65–0.75. It is not clear whether this estimate would be valid, if the asperity was broken by the source rupture rather than in a spontaneous manner. A numerical experiment [15] has shown that in an inhomogeneous fault model involving a set of three prestressed asperities, the spontaneous rupture of the first asperity occurred at a velocity much smaller than c_S, that of the second was more rapid, and the third ruptured at a velocity significantly above c_S. We will use the value $\nu_a = 1.35$ in our calculations (the adopted rupture velocity is $0.78\ c_p = 1.35\ c_S$).

The most serious idealization in [13] is the assumption of an infinite fault (crack). The Rayleigh waves excited by the rupture of an asperity then propagate away to infinity, and it is only the displacement step mentioned above that is radiated inward in the shape of body waves. The more realistic case, a circular fault with a circular asperity at the center, was considered in [14]. Surface waves will in that case be diffracted at the fault edges with the result that the wave shape will be altered (case B in Figure 1): the displacement step ends in a smooth return to zero during a time of the order of $2R_c/c_R$, where R_c is the fault (crack) radius and c_R is Rayleigh wave velocity. Also important is the case of an asperity located near the edge of the fault. The expected diffraction-affected pulse shape for that case is also shown in Figure 1.

It should be noted that the amplitude of an acceleration pulse due to a single asperity is not affected by the asperity size, the stress drop $\Delta\tau$ alone being of importance. We shall take up the simplest case of a circular asperity for which $F_0 = \pi R_a^2 \Delta\tau$ and assume $f(t) = 1-\cos(2\pi t/T)$ as the shape of velocity pulse. We then easily find that, for this pulse, $\ddot{F}_{0\ max} = \max|F_0(t)| = 2\pi F_0/T^2$. Putting $T = T_a$ in this expression and substituting $\ddot{F}_{0\ max}$ into (2) for $u \equiv a$, we find the peak acceleration in SH motion as

$$a_{\max}^{SH} = \left(\frac{\pi D^{SH} v_a^2}{8} \right) \frac{\Delta\tau}{\rho r}. \tag{4}$$

We now modify this estimate to be able to use it for the interpretation of observed near-source acceleration. To do this, we first replace D^{SH} with

$$D = \left(\frac{1}{2} \left(\frac{1}{4\pi} \int_{\Omega} (D^{SH})^2 \, d\Omega + \frac{1}{4\pi} \int_{\Omega} | D^{SV} |^2 \, d\Omega \right) \right)^{1/2}, \tag{5}$$

where Ω is the unit sphere. According to our rough estimate, $D \approx 1.16$, $D_* = 2.6$. It should also be remembered that amplitude will be twice as large at the free surface. The modified (4) will thus hold for a uniform half-space. The difference in the impedances ($c_S\rho$) at the source and beneath the station for typical frequencies of 3-10 Hz will enhance the amplitude by another factor of two [2]. The final formula for $v_a = 1.35$ is

$$a_{max} = 3.3\Delta\tau/\rho r. \tag{6}$$

A similar expression (with the constant equal to 1) was derived in [21] through physical reasoning.

We now turn to the spectra of the radiated waves (see Figure 1). The characteristic frequencies f_a and f_c are controlled by the rise time T_a and the pulse duration T_c:

$$f_{a,c} = C_B/T_{a,c}, \tag{7}$$

where $C_B = 0.8$ will be used in subsequent calculations; this value was obtained for a symmetrical trapezoidal pulse whose rise time is 20 percent of total duration. Case A at frequencies $f \ll f_a$ yields the following amplitude spectra of velocity and acceleration:

$$\begin{gathered} \dot{u}(f) = u(t)|_{t=\infty} = DF_0/4\pi\rho c_S^2 r, \\ a(f) = 2\pi f \dot{u}(f). \end{gathered} \tag{8}$$

These formulas can also be used for case B, approximately within the frequency range of $3f_c < f < f_a/3$. The level of velocity spectrum (or acceleration spectrum at a fixed frequency) is thus controlled by the "seismic force" F_0. For the vicinity of f_a, we write the acceleration spectrum (by analogy with the Brune model) in the form

$$a(f) = \left(\frac{DF_0}{2\rho c_S^2 r}\right) \frac{f}{1 + (f/f_a)^{\nu_a}}, \tag{9}$$

with $\nu_a = 2$ as given by the observations [17]. The peak acceleration spectrum is then

$$a(f)_{\max} = \frac{DF_0 f_a}{4\rho c_S^2 r} \tag{10}$$

Hence $a(f)_{max}$ is proportional to $F_0 f_a$ (or $\Delta\tau R_a$). Figures 1 and 3 (see below) show a smooth peak given by (9) represented schematically as an angle.

The concentration of stresses over an asperity is important in terms of the mechanics of faulting. Das and Kostrov [13] assume the asperity to be in welded contact, which involves an integrable stress singularity at its edge. It can be supposed that actual asperities "plastically adapt themselves" to loading and that the distribution of stress over an asperity is not dramatically nonuniform. An analysis of the asperity modeled by an elastic paraboloidal hill in contact with an elastic plane lends support to this supposition. This problem is identical with the well-known Hertz problem of elastic balls in contact [1]. In the case of contact without friction, it predicts an ellipsoidal distribution of normal stress (similar to that of displacement contrast for an elliptical fault loaded at infinity). It may well be assumed that the incorporation of dry friction and shear stress would produce a similar distribution of shear stress (the stress increasing inward rather than outward). A detailed analysis of this case is hampered by the necessity of incorporating the load history. We also note that an asperity in welded contact, as in [13], corresponds in this context to a "flat-topped hill" with vertical sides.

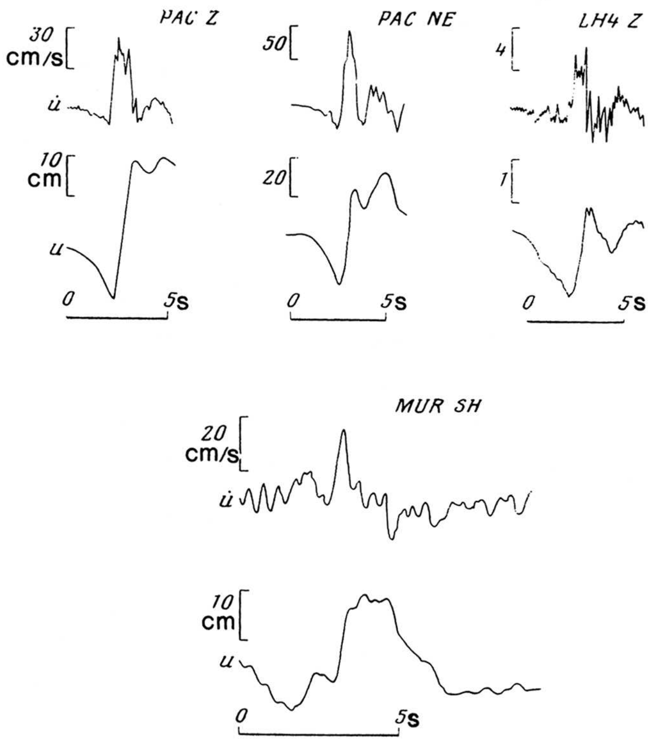

Fig. 2. Signals supposed to have been radiated by asperities. Top — an asperity in the lower portion of the San-Fernando earthquake source. The integrated accelerograms were recorded at Pacoima (two components) and at Lake Hughes-4 (one component) (after [24]). Bottom — same for the 1968 Tokachi earthquake and seismic station Muroran (after [29], pulse 2). In all cases one can see a unidirectional velocity pulse and a displacement step. The latter may have been distorted by double integration.

The above reasoning suggests that, even though the theory of [13] involves a singularity, the assumption of mean stress drop $\Delta\tau$

over an asperity is admissible and, moreover, $\Delta\sigma(x, y)$ can be assumed, for estimation, to be constant over the asperity area.

Figure 2 presents two examples of recorded radiation due to the rupture of a single asperity. The records have a characteristic feature — a unidirectional pulse of velocity. Both examples were obtained from an analysis of the accelerograms recorded during large earthquakes, which may contain many pulses of this type. A single pulse can be visible either when it is one of few, or when it has an abnormally large amplitude. The examples illustrate both of these possibilities. The first example (Figure 2, top) has been mentioned in [21] (we borrowed it from [24]). A remarkably clear-cut velocity pulse suggests that the idea of a crack between the sides of a fault may be more than a mere idealization. The other example (Figure 2, bottom) is borrowed from [29]. Although the authors interpreted the observed pulse as being due to a subevent (crack), our interpretation does not seem to be less substantiated. We used the parameters of the pulses to evaluate F_0, R_a and $\Delta\tau$ for both cases (Table I).

SEISMIC RADIATION FROM A FAULT WITH MANY ASPERITIES

We saw in the preceding section that the main pulse of velocity and acceleration due to a single rupturing asperity is controlled by its "local" properties — stress drop and size. We will assume therefore that the case of several asperities experiencing rupture can be treated by calculating the radiation of each using the formulas for a single asperity and summing the contributions.

Consider a fault model as a crack whose sides contact in many small patches or asperities. Proceeding in the spirit of Kostrov's book [4], we assume an earthquake source located on such a fault to be a shear crack, when viewed "macroscopically", for instance, a circular crack of radius R_c with a constant stress drop $\Delta\tau$, its formation being associated with a "macroscopically" smooth propagation of the rupture front. Viewing the picture "microscopically", we see a "wave" of individual asperity ruptures instead of a smooth motion of the front. We will name the source model with many asperities the MA model. The value of $\Delta\sigma$ in this model can be unambiguously defined only for that portion of the fault plane where many asperities are present. The assumption of $\Delta\sigma$ constant over the fault plane means that the mathematical expectation of the quantity thus defined is the same for any area. The relation between the macroscopic $\Delta\sigma$ and the mean stress on the asperity $\Delta\tau$ for a given source is found from a balance of forces in macro- and microrepresentation, yielding for identical asperities

$$S\Delta\sigma = \sum_i F_{0,i} = N_S \Delta\tau S_a, \tag{11}$$

where S is the crack area, N_S the number of asperities on it, $\Delta\tau$ and S_a being treated as means over a set of asperities.

We now introduce an important parameter, a "space factor" k_{sp} which is equal to the portion of the fault plane occupied by asperities. Then the number of asperities on the fault plane can be expressed as

$$N_S = k_{sp} S/S_a = k_{sp} R_c^2/R_a^2, \tag{12}$$

the last expression being true for a circular crack of radius R_c and identical circular asperities, the case with which we shall be concerned. Expression (11) now yields a relation between $\Delta\sigma$ and $\Delta\tau$

$$\Delta\tau = \Delta\sigma/k_{sp}. \tag{13}$$

This relation which was derived from physical reasoning is in satisfactory agreement with the results obtained in [10] where the mean seismic moment due to some individual asperity (a single or one of many) located at random within a circular fault of radius R_c is evaluated from (in our notation)

$$M_{01} = (16/7)R_a^2 R_c \Delta\tau = (16/7\pi)F_0 R_c. \quad (14)$$

Considering that $M_{01} = M_0/N_S$ and $M_0 = (16/7)R_c^3\Delta\sigma$, [14] readily gives (13). Finally, we need to introduce the mean distance between the asperities

$$d = (S/N_S)^{1/2} = (\pi/k_p)^{1/2} R_a. \quad (15)$$

Now assume that the asperities rupture at random times. Then for a source with N_S identical asperities, the amplitude acceleration spectrum is

$$a(f) = a_1(f)N_S^{1/2}, \quad (16)$$

where $a_1(f)$ is that due to a single asperity.

Note that, after the rupture front has traveled along the fault, ground motion can in principle occur with sticking with one and the same asperity multiply radiating SP pulses. If this is the case, (16) is not valid and the number of pulses is proportional to ST_c rather than to S, where T_c is the duration of the source process. The usual similarity assumptions would give $a(f) \sim M_0^{1/2}$ in our version, whereas we have $a(f) \sim M_0^{1/2}$ in the case under consideration, M_0 being the seismic moment. Earlier [2] we assumed the latter version, while the hypothesis of a constant "effective" $\Delta\sigma$ [22] implies the former. Note that the observed short-period spectral trends reported in [2] indicated $a(f) \sim M_0^{0.36}$, whereas more recent data suggest $a(f) \sim M_0^{0.47}$. Observations of this kind do not therefore provide decisive constraints. Recalling the results from [33], where a "seismic antenna" was successfully used to monitor the motion of a "bright" high-frequency radiator at the rupture front, we are inclined to acknowledge the first version (an asperity radiates only once, $a(f) \sim M_0^{1/3}$), explaining away the departure of the observed trends from 1/3 as the effect of secondary factors.

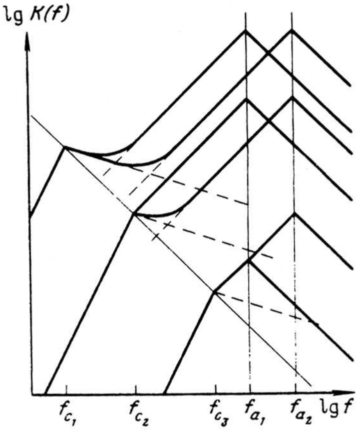

Fig. 3. Modified source spectra $K(f) \equiv f^2 M_0(f)$ for the MA model on the assumption that the main smooth source (crack) obeys similarity requirements. The plots are given for three variants of M_0 and f_c ($f_c \sim M_0^{-1/3}$, $\Delta\tau$ = constant) and two variants of f_a. The stress drop produced by asperity rupture is assumed to be identical in all cases. The lowermost line shows the spectrum for a weak earthquake.

Let us consider the general structure of a source spectrum in the MA model. We will assume that a "macroscopic" crack radiates a smooth displacement pulse having an $\omega^{-\gamma}$ spectrum with γ = 2.5-3.5 and a corner frequency $\omega_c = 2\pi f_c$. Superposed on this pulse is the radiation due to asperities whose f_c is the same as that of the "macroscopic" pulse. It should be emphasized that in the MA model the number of asperities, and hence the total energy of SP radiation, increases with the size of the source in direct proportion to the source area rather than to ST_c as was assumed in [2]. That means that, when considered at macrolevel, SP radiation energy can be regarded as "quasithermal" losses [4] or as rupture energy.

In [2] we proposed that the source spectrum $|\dot{M}_0(f)|$, which is the modulus of the spectrum of $\dot{M}_0(t)$ describing the rate of change of the seismic moment of an equivalent point source should be replaced by the function

$$K(f) = f^2\left|\dot{M}_0(f)\right| \tag{17}$$

which describes the acceleration spectrum. For a uniform medium

$$a(f) = \frac{(2\pi)^2 RK(f)}{4\pi\rho c_S^3 r} \tag{18}$$

Figure 3 presents several schematic spectra K(f) for our model, including the hypothetical spectrum of a weak earthquake which is represented by the rupture of a single asperity as conceived in the model. The MA model obviously yields spectra with the similarity principle violated, the character of the violation being consistent with the modified model B [6] and the system of spectra in [2].

One can use (8), (9), (16) and (18) to determine the level of K(f) in the MA model containing identical asperities. We have for the low-frequency fall-off between f_c and f_a

$$K(f) = 0.5D_* c_S k_{sp}^{1/2} R_c R_a \Delta\rho\tau f \tag{19}$$

and for the vicinity of the K(f) peak (i.e. near f_a), putting $\nu_a = 2$ and finding from (4) and (7)

$$R_a = \nu_a c_S C_B / 2fa, \tag{20}$$

we arrive at

$$K(f)_{max} = K(f_a) = 0.125 D_* c_S^2 k_{sp}^{1/2} C_B \nu_a R_c \Delta\tau. \tag{21}$$

The quantity D_* is here to be understood as an rms value over a sphere and so is K(f).

The formulas derived in this section can be used along with observational data to determine various parameters of the model. We will use spectral information from [32] and [34]; specific levels of $K(f)$ and f_a will be for the value $\log M_0$ (dyne·cm) = 26 (M_L = 6.6). In that case, using $\Delta\sigma$ = 30 bars as a typical value, we get $R_S = (16/7) M_0/\Delta\sigma^{1/3}$ = 11.4 km. We also take the following values for all subsequent estimates: $D = 1.16$, $D_* = 2.6$, $C_B = 0.8$, $\nu_a = 1.35$, c_S = 3.5 km/s, and $\rho = 2.7$ g/cm^3. The mean spectra from [34] can be used to find the peak frequency of the observed spectrum $K(f)$ for M_L = 6.6; treating it as f_a, we get f_a = 2.4 Hz, and (20) yields R_a = 0.8 km. Using k_{sp} = 0.1 as a trial value for this parameter, we obtain from (13) the estimate $\Delta\tau_1$ = 300 bars. The values of $K(f)$ will be used for $f = 0.63$ Hz $\ll f_a$ and for $f = f_a$ = 2.4 Hz. the data from [32] and [34] when transformed to the case of a uniform crust [2] yield $\log K(0.63) = 24.30$, $\log K(2.4) = 24.52$. Solving (19) and (21) for $\Delta\tau$, we get the estimates $\Delta\tau_2$ = 250 bars and $\Delta\tau_3$ = 210 bars.

STATISTICS OF $\Delta\tau$ AND PROPERTIES OF ACCELEROGRAMS

In this section we will derive an estimate for $\Delta\tau$ on the basis of observed near-field peak accelerations. To do this, we will first assume the peak acceleration in an accelerogram to be formed by the pulse due to an asperity rather than a fluctuation of a sum of random terms. This assumption will be substantiated below.

A set of asperities can be thought of as a statistical ensemble having the distribution function

$$P(\Delta\tau' < \Delta\tau,\ R_a^1 < R_a) = F(\Delta\tau,\ R_a).$$

Below we shall only consider the distribution over $\Delta\tau$. We recall that $\Delta\tau$ controls the peak acceleration in the pulse radiated by an asperity. The "heavy-tailed" character of breaking-strength distribution for realistic models of actual earthquake sources [28] and the shapes of near-field accelerograms, prescribe the employment of a power-law rather than lognormal distribution:

$$P(\Delta\tau' > \Delta\tau) = (\Delta\tau/\Delta\tau_{min})^{-\alpha}. \tag{22}$$

We now find for this distribution the median of the largest value in a sample of N asperities. A simple calculation yields

$$\Delta\tau_{N;0,5}/\Delta\tau_{min} = (1 - 2^{-1/N})^{-1/\alpha} \approx (1{,}45N)^{1/\alpha}. \tag{23}$$

The mean and variance for (22) are

$$\langle \Delta\tau \rangle = \left(\frac{\alpha}{\alpha - 1}\right) \Delta\tau_{min}, \tag{24}$$

$$\langle (\Delta\tau - \langle \Delta\tau \rangle)^2 \rangle = (\alpha/(\alpha - 1)^2 (\alpha - 2))\, \Delta\tau_{min}. \tag{25}$$

We will use these results to interpret near-field peak acceleration data: we assume that the observation site is at the earth's surface at the distance r_0 from the plane of a roughly vertical fault where the earthquake occurs. Peak acceleration can be determined by taking into account the contribution of the nearest portion of the fault plane, because radiation due to more distant points of the fault will not produce a peak value. That means that the dependence on source dimensions (or magnitude) will only take place for those comparatively small shocks whose sources cannot entirely cover the nearest fault portion mentioned and that the overpassing of a critical value will make the dependence vanish (cf. [11]). Assuming a direct relation of a source area to magnitude, one can determine the critical magnitude from the area of that portion. To make numerical evaluation possible, we take this area to be the lower half of a vertical square with the side $2r_0$ whose center is at the earth's surface at the point of the fault which is nearest to the observation site. The area in question is $2r_0^2$ and the mean distance between a random point in the area and the receiver is approximately $r = 1.2r_0$.

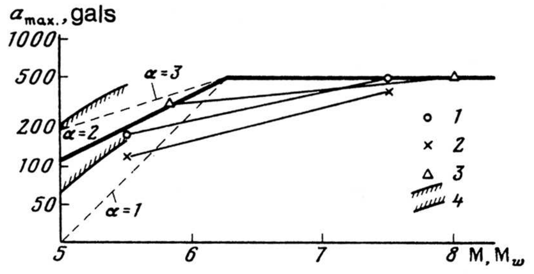

Fig. 4. Experimental relationships of peak horizontal acceleration a_{max} versus magnitude and theoretical broken lines for distance r_0 = 10 km to the fault for a power-law distribution with α = 1, 2 (preferred value), and 3: 1 — after [16]; 2 — after [11] (both plots are for 10 km distance to the nearest point of the vertical projection of the fault on to the earth's surface); 3 — after [19] for epicentral zone; 4 — region of data points after [17] for a hypocentral distance of 10 km

Let us take the specific value r_0 = 10 km; then the area of the nearest portion is S = 200 km^2 for which the critical magnitude is M 6.3. For smaller magnitudes, the median peak acceleration a_{max}, proportional to $\Delta\tau_{max}$ (6), decreases with decreasing magnitude owing to a smaller number of asperities N in (23). As N~S, we have

$$\log a_{max} \approx \frac{1}{\alpha} \log S + \text{constant};$$

since log S≈M + constant, this gives a relation between a_{max} and M. Figure 4 presents observed $a_{max}(M)$ relations based on different sets of data and a fitting broken line whose rising portion is plotted in accordance with the value α = 2. It is apparent that α = 2 is a rough estimate, but the values α = 1 or α = 3 are obviously much less likely. The broken line takes into account the fact that the observed relationships in [11, 16] correspond approximately to the distance to the nearest point of the fault (in particular, to its edge), while our estimates are approximately relevant to distances to the center of the fault. Since the observations and the theoretical broken line are roughly consistent, this indicates an approximate validity of the underlying theory. The level of the broken line for large M (a_{max} = 500 cm/s^2) can be used to evaluate $\Delta\tau$ from (6) on the assumption that the peak acceleration has been caused by an asperity. Reversing (6) for a random location of the strongest of N asperities, we find the stress drop for it as

$$\Delta\tau_{max} = 0{,}30\,\rho\bar{r}a_{max}. \qquad (26)$$

Assuming that (26) gives $\Delta\tau_{N;0.5}$, we derive the mean $\Delta\tau$ from (23) and (24):

$$\left\langle\Delta\tau\right\rangle = \frac{\alpha}{\alpha-1}\,(\pi R_a^2/1.45\,k_{sp}\,(2r_0^2)^{1/\alpha}\Delta\tau_{max}. \qquad (27)$$

Putting $\bar{r}$ = 12 km, α = 2, and a_{max} = 500 cm/s^2, we use (26) and (27) to obtain another empirical estimate $\Delta\tau_4 = \left\langle\Delta\tau\right\rangle$ = 260 bars.

Assuming $\left\langle\Delta\tau\right\rangle$, R_a, and k_{sp} to be constant in (26) and (27), we get the relation

$$a_{max} = f(\alpha)\,r_0^{\frac{2}{\alpha}-1}. \qquad (28)$$

To sum up, peak acceleration for α = 2 around the middle of the fault (at distances smaller than the fault width) is independent of the distance to the source. This theoretical result is in approximate agreement with macroseismic observations, which is another evidence in favor of the assumption α = 2.

We shall now be concerned with the question of how peaks form in an accelerogram according to our model. This is of interest, in particular, in relation to the question of whether an accelerogram can be treated as a quasistationary Gaussian process (as is often done).

We will begin by deriving some general results and then examine what can be expected for the distribution given by (22) with $\alpha \approx 2$.

Consider a rectangular source of the dimensions L×W with rupture propagating along the source length (L) at the velocity $v = \nu_c c_S$, so that the rupture time is $T_c = L/v$. The number of asperities ruptured during the time T_a will then be

$$m = (N/T_c)T_a = 2k_{sp}\nu_c W/\pi\nu_a R_a = 4k_{sp}\nu_c W f_a/\pi\ \nu_a^2 C_B c_S. \qquad (29)$$

The pulses obviously do not superpose when $m \lesssim 1$, at least near the source, and the accelerogram will have an unusual appearance. At greater distances it will approach a quasistationary Gaussian process owing to scattering and multipath propagation. The value m = 1 gives the following critical source width:

$$W_{cr} = \pi\nu_a^2 C_B c_S/4k_{sp}\nu_c f_a. \qquad (30)$$

When $\nu_{sc} = 0.6$, we have $W_{cr} = 30$ km. That means that a multiple superposition of pulses can only occur during very large earthquakes.

Even then, however, the near-source accelerogram will not resemble a Gaussian process when $\alpha \approx 2$. As can be seen from (25), distribution (22) has infinite variance when $\alpha \approx 2$. That means that, even with multiple superposition of pulses, the main contribution to the amplitude is made by some specific asperity. This justifies our assumption that the peak acceleration is due to a single asperity. If one is allowed some exaggeration, one can speak of each positive spike in an accelerogram as being due to a specific asperity. Obviously, the results of this section can be used for modeling accelerograms.

STATISTICS OF $\Delta\tau$ AND BARRIERS ON THE FAULT

The problem of the statistics of $\Delta\tau$ can also be approached theoretically by following the assumption of Fukao and Furumoto [18] that an earthquake source develops in an approximately self-similar manner: a source passes through a sequence of stages so that its current dimensions ought to form a geometrical progression: L_0, bL_0, b^2L_0, ... Each stage consists in overcoming a "barrier" located around the perimeter; the barrier is passed over in a near-critical regime with a high probability of stoppage. Barriers of different strengths form a hierarchical system of closer and closer spaced grids on the fault (the larger the spacing, the stronger the barrier). At each stage the source occupies one square (or several squares) in the grid of the relevant rank.

We will assume that barriers in this model are chains of strong asperities spaced at a uniform interval d. We will take a fixed region of the fault having an area S^*. The total length of the grid units with the square side q will be found as (number of squares)$\times$(semiperimeter of square) $\approx (S^*/q^2)\times 2q = 2S^*/q$. In that case the number of asperities in this grid is

$$N^*(q) = 2S^*/dq. \tag{31}$$

It has been assumed that the larger the grid spacing, the stronger the asperities. Supposing for the sake of definiteness that we have $\Delta\tau = \Delta\tau_b \sim q^\beta$ for the asperities, we substitute $q \sim \Delta\tau_b^{1/\beta}$ into (31) and get

$$N^*(\Delta\tau_b) \sim S^*/\Delta\tau_b^{1/\beta} \tag{32}$$

In order to relate q and $\Delta\tau_b$, we will use the basic relationship of the theory of a crack with an end zone:

$$\Delta\sigma_{coh} \approx (R_c/a)^{1/2}\Delta\sigma, \tag{33}$$

where $\Delta\sigma_{coh}$ is cohesion less friction in the circular end zone of the width a, and $\Delta\sigma$ is the stress drop on the fault plane of radius R_c. A "microscopic" version of this criterion can be derived by taking into account that the spacing between the barriers is d and correlating the distance between the "rows of asperities", also d, with a. The requirement of the equality of forces gives $\Delta\sigma_{coh}d^2 = S_a\Delta\tau_b = d^2k_{sp}\Delta\tau_b$, the breaking criterion for a barrier unit being obtained as

$$\Delta\tau_b = (R_S/d)^{1/2}k_{sp}^{-1}\Delta\sigma. \quad (34)$$

Assume that a barrier breaks when the mean $\Delta\tau_b$ for the chain is equal to the critical value. In that case, treating (34) as relating the barrier grid size $q \approx 2R_S$ to the $\Delta\tau_b$ for the asperities that make up the barriers, we find that $\beta = 0.5$ in (32). The probability of different $\Delta\tau_b$ is then

$$P(\Delta\tau_b) \sim 1/(\Delta\tau_b)^2.$$

The values q, as well as $\Delta\tau_b$, form a geometrical progression. Then

$$P(\Delta\tau \geqslant \Delta\tau_b) \sim 1/(\Delta\tau_b)^2 + 1/b(\Delta\tau_b)^2 + 1/b^2(\Delta\tau_b)^2 + \ldots$$
$$\ldots = 1/(\Delta\tau_b)^2(1/1-b^{-1}) \sim 1/(\Delta\tau_b)^2, \quad (35)$$

where the accidental parameter b is the ratio of the q progression.

We do not wish to assert that all strong asperities with some $\Delta\tau$ form barrier chains. It would be sufficient to assume that the portion of asperities with some fixed $\Delta\tau$ that form barriers is independent of $\Delta\tau$. We can then compare (35) directly with (22) and provide additional confirmation of our power-law distribution of $\Delta\tau$ with $\alpha = 2$.

We have implied that the mean stress drop $\Delta\sigma$ is the same at every stage of the source evolution. This follows from our fault model and, at the same time, is consistent with $\Delta\sigma$ being independent of L for actual sources each of which can be treated, after Fukao and Furumoto [18], as a "frozen" phase in the evolution of a larger source.

We are going to use the above approach to construct an estimate for the mean $\Delta\tau$. We will assume that the subset of N_p asperities which constitute the portion of the barrier perimeter that is close to rupture (or ruptures under near-critical conditions) itself obeys (35). For our estimation we will assume this portion to be a half of the perimeter. For a circular fault we have $N_p = \pi R_S/d$. For the mean of the N_p values of $\Delta\tau$, or $\Delta\tau_p$, (34) holds. For the strongest of the N_p asperities we can use (23) and (24) to derive the median $\Delta\tau_M$:

$$\Delta\tau_M = (\Delta\tau_p/2)(1.45N_p)^{1/2}. \tag{36}$$

The counterpart of this formula

$$\Delta\tau_M = (\langle\Delta\tau\rangle/2)(1.45N_S)^{1/2} \tag{37}$$

relates $\Delta\tau_M$ to the mean value of $\Delta\tau$ for the entire source, provided we make the plausible assumption that the strongest of N_S asperities is located at the perimeter. Combining these results, we get

$$\langle\Delta\tau\rangle = \Delta\tau_p(N_p/N_S)^{1/2} = \Delta\sigma/k_{sp} \tag{38}$$

which is identical with (13). To sum up, we have not derived a new estimate for $\Delta\tau$, but then we have demonstrated that our model is internally consistent in this respect.

DISCUSSION

The MA model has yielded several estimates for mean stress drop on the surface of an asperity: $\Delta\tau_1$ = 300 bars based on a "global" $\Delta\sigma$ and a given k_{sp}, $\Delta\tau_2$ = 250 bars based on the low-frequency slope in the acceleration spectrum, $\Delta\tau_3$ = 210 bars based on the peak of an acceleration spectrum, and $\Delta\tau_4$ = 260 bars based on near-source peak accelerations. Although all estimates are in reasonable agreement, one should not forget that the important dimensionless parameters k_{sp} and ν_a have been assigned nearly a priori values. Note that all three formulas for $\Delta\tau_{2-4}$ (which have not been written out in explicit form) include these two parameters in the same combination $k_{sp}^{-1/2}\nu_a^{-1}$, while the formula for $\Delta\tau_1$ contains $K_{sp}^{-1}\nu_a^0$. That means that the consistency will not deteriorate, if k_{sp} and ν_a are varied together so as not to affect the combination of $k_{sp}\nu_a^{-2}$ whose value must equal 0.055 (=0.1×1.35^2). The reasonable range for ν_a is 0.6 to 1.5 which yields the range 0.02 to 0.125 for k_{sp} and 1500 to 250 bars for $\Delta\tau$. Both ranges look more or less admissible, yet the value as high as 1500 bars for the mean $\Delta\tau$ and the value as low as 0.02 for k_{sp} seemed to be unlikely. Our final estimates are: k_{sp} = 0.04-0.125, ν_a = 0.85-1.5, $\Delta\tau$ = 750-250 bars, and R_a = 0.6-0.9 km.

A few more points seem to be worthy of consideration. Agreement has been found between the estimates of the stress drop distribution function for asperities based on $a_{max}(M)$, on the approximate constancy of a_{max} near the fault, and on the considerations related to the source dynamics.

Should our conclusion about the α = 2 power law for $\Delta\tau$ and for accelerogram peaks find corroboration, that would mean that the use of the quasistationary Gaussian approximation to describe near-field accelerograms is not entirely correct. It would then be reasonable to use an alternative procedure for describing and modeling the accelerogram on the basis of the present model by superposing comparatively infrequent pulses due to specific asperities.

We have not touched upon the important problem of how asperities are distributed by size. The fact that the estimate $\Delta\tau_3$ based on the spectral peak is considerably lower than the estimate $\Delta\tau_2$ based on the low-frequency radiation indicates that, as might be expected, asperities are not quite of the same size. This conclusion can also be reached from a comparison of the size estimates in Table I with the mean size. Nevertheless, the presence of a more or less pronounced peak in the spectra of many accelerograms permits us to regard a source model with asperities of the same size as a reasonable first approximation.

CONCLUSIONS

1. It is proposed that the rupture of an asperity be considered as a typical subsource radiating a short-period pulse from the source of a large earthquake and as the principal component of a small earthquake souce.

2. The results of S. Das and B.V. Kostrov were used to put forward a simple theory for seismic radiation from a single asperity and from a set of many asperities. The seismic force of an asperity F_0 was defined as the basic parameter which controls the low frequency spectral branch of seismic radiation.

3. The experimental data on near-field peak accelerations and on the level of far-field spectra are consistent and indicate a typical stress drop at an asperity, $\Delta\tau$, of the order of a few hundred bars.

4. Evaluation of barrier (asperity) strength combined with the Fukao-Furumoto hypothesis of stepwise self-similar fault evolution yielded a power law, "heavy-tailed" distribution of asperities according to strength. It agreed with experimental data on peak accelerations.

5. It is proposed to describe and model the accelerogram as a sum of poorly overlapping pulses due to individual asperities that obey a power-law distribution of the parameter $\Delta\tau$.

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